The Validity of the Uniform Mixing Assumption: Determining Human Exposure to Environmental Tobacco Smoke*

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Abstract

When using the mass balance equation to model indoor air quality, the primary assumption is that of uniform mixing by which a given point in a single compartment is assumed to have the same instantaneous pollutant concentration as all other points. Although this assumption may be unrealistic, predictions (or measurements) of exposures at single points in a room may be within acceptable limits of error (e.g., 10%) under certain conditions. In this paper, three studies of the mixing of environmental tobacco smoke (ETS) pollutants are reviewed, and data from several other ETS field studies are presented. Under typical conditions for both short sources (e.g., 10 min) and the continuous sources of ETS in smoking lounges, I find that average exposure concentrations for a single point in a room represent the average exposure across all points in the room within 10% for averaging times ranging from 12 to 80 minutes. I present a method for determining theoretical estimates of acceptable averaging times for a continuous point source.

Key Words: Mixing of Pollutants, Ideal Mixing, Uniform Mixing, Environmental Tobacco Smoke, The Mass Balance Equation, Exposure Assessment, Exposure Modeling

Abbreviations: ach, air changes per hour; CO, carbon monoxide; ETS, environmental tobacco smoke; RSP, respirable suspended particles

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1 Introduction

When exposure measurements for environmental tobacco smoke (ETS) or other air contaminants are unavailable or potentially unrepresentative, exposure estimates can be made using mathematical models. As discussed in another article in this volume (1), estimates of population exposure require both the time spent being exposed and the magnitude of exposure (e.g., the average pollutant concentration). In many studies, the mass balance equation has been the method of choice to describe indoor air pollution, and it has been repeatedly applied to the indoor pollutants present in ETS ($2-5$). An article in this volume (6) presents the mass balance equation (including some historical background) and shows how it is applied to the modeling of ETS.

One of the primary assumptions made in the application of the mass balance equation is that of uniform, or ideal, mixing. Each point in a room is assumed to have the same instantaneous pollutant concentration as all other points. Another common assumption (based on the assumption of uniform mixing) is that time-averaged concentrations measured at a single point in a room represent either the time-averaged concentrations at any arbitrary point in the room or the time and spatially-averaged room concentration. But how accurate is such an assumption when making estimates of human exposure to ETS? Can exposures be assigned to specific occupants of a space based on a single spatially-localized estimate for the space?

This paper reviews information relevant to these questions. Three recent papers have focused on the mixing of indoor air pollutants. Mage and Ott (7) have introduced a standard temporal breakdown for ETS studies, Baughman et al. (8) have measured the time required for the mixing of a nearly-instantaneous pollutant emission in a controlled chamber under conditions of natural convection, and Drescher et al. (9) have measured mixing times for conditions of forced convection. In the present work, these papers are discussed (Section 2) and analyses of data from other studies are presented (Section 3), which give insight into the validity of using the mass balance equation to predict ETS exposures for short and temporally continuous point sources in both occupational and other settings. Section 4 presents the results of original calculations that show how, given the mixing time for a nearly-instantaneous source, we can estimate the required averaging time for a continuous source (under otherwise identical conditions) such that single-point exposures represent the room average within about 10%.

2 Previous Studies of Uniform Mixing

Mage and Ott (7), in discussing the mass balance equation, suggest that models using an exponential mixing factor that is less than 1 (thereby reducing the theoretical removal rate) should not be used to make estimates of human exposure to air pollutants. Instead, each location should be examined to determine the degree of non-uniform mixing, and if mixing is found to be unacceptably nonuniform, a multi-compartment model should be used with a mixing factor of 1 for each compartment. In the process of examining the degree of mixing for a given location, Mage and Ott propose the delineation of three sequential time segments that can occur during the study of a single, short source: (1) the time τ_α during which the source is active; (2) the time τ_β during which the source is off and the room is not well-mixed; and (3) the time τ_γ during which the source is off and the room is well-mixed. The following is a representation of these three time segments:

$$|--\tau_\alpha \ --||----\tau_\beta \ ---||------\tau_\gamma \ ------|$$

where τ_α = *source-on-unmixed* time segment, τ_β = *source-off-unmixed* time segment, and τ_γ = *source-off-mixed* time segment. I use this notation in the current paper. The time $t = 0$ is taken to be the time at which the source begins.

For cigarettes, τ_α, which is the time for which the cigarette is actually being smoked, is typically 6 to 11 min. The amount of time that is required for the room to become well mixed after the source stops, τ_β, is dependent on factors such as the presence of (1) mechanical flow devices, (2) air conditioning, (3) heating equipment, (4) sunlight, and (5) active persons. The value of τ_β also depends on the location of the source(s) in the room and whether or not it(they) is(are) stationary. Each of these factors could have a large effect on τ_β. For example, turbulent air flow created by moving persons, heat input, or fans would tend to decrease τ_β. Although we cannot readily determine an exact magnitude of the effect, τ_β can be measured for a variety of conditions in typical locations such as offices, lounges, taverns, homes, and vehicles. The third time segment, $\tau_\gamma)$, extends from the end of the second time segment, τ_β, until such time as the room concentration $z(t)$ decays exponentially to approximately 1% of the peak concentration. After this point, the pollutant concentration is considered undetectable.

The peak concentration is taken to occur at the end of the τ_α time period. Assuming the pollutant concentration decays exponentially after the end of the τ_α time period, the time it takes to reach 1% of the peak concentration is $-\tau$ ln(0.01) = 4.6 $\tau \approx 5\ \tau$, where τ is the residence time (the time it takes to reach $\frac{1}{e}$ times the original concentration). Therefore, the approximate time that elapses between the end of the τ_α period and the end of the τ_γ period is 5 τ .

A crucial quantity in assessing exposure is the time-averaged pollutant concentration. As Mage and Ott demonstrate, the overall pollutant concentration at a single point in a room z^{ave} can be broken down into a weighted average of the pollutant concentration in each of the three time periods described above:

$$z^{ave} = z_\alpha^{ave}\tau_\alpha/T + z_\beta^{ave}\tau_\beta/T + z_\gamma^{ave}\tau_\gamma/T \tag{1}$$

where $T = \tau_\alpha + \tau_\beta + \tau_\gamma$ is the total study duration, z_α^{ave} is the average concentration during the τ_α interval, z_β^{ave} is the average concentration during the τ_β interval, and z_γ^{ave} is the average concentration during the τ_γ interval. If the percentage of time during the transition period from the poorly-mixed to the well-mixed state, τ_β, is small compared to the total source-off time period, $\tau_\beta + \tau_\gamma$, then the proportion contributed to the overall average is small for the middle term on the right side of the above equation, and z^{ave} is probably a good approximation of the well-mixed concentration.

Baughman et al. (8) have determined total mixing times[1] ($\tau_{mix} = \tau_\alpha + \tau_\beta$) in a chamber under different conditions of natural convection after the release of a pollutant from a nearly instantaneous source (τ_α = 6 min): (1) quiescent air (τ_{mix} = 80−100 min); (2) the presence of a 500-W water heater (τ_{mix}= 13−15 min); and (3) the presence of incoming solar radiation (τ_{mix}= 7−10 min). The chamber was considered to be well-mixed when the relative standard deviation (standard deviation divided by the mean) over 41 points in the room was 0.10 or less. Since the air exchange rate remained at about 0.03−0.08 air changes per hour (ach) (τ = 12.5−33.3 hrs) for all the experiments, the well-mixed time period τ_γ as defined by Mage and Ott is very long (more than 24 hrs) – regardless of the value of τ_β. As a result, the proportion of time spent mixing is only 0.01−2.5% of the total source-off time ($\tau_\beta + \tau_\gamma$). Thus, the uniform mixing criteria of Mage and Ott is met. However, since air exchange rates in American homes are usually in the range of around 0.5 to 2 ach (13), a more realistic source-off time might be 2.5 to 10 hours. In this case, the criteria of Mage and Ott may not be met. We also need to consider that Americans typically spend less than 8 hours being exposed to ETS in a given location (1).

To study the effect of shorter averaging times on errors in exposure estimates, an exposure index has been introduced by Baughman et al. The exposure index is defined as the time-averaged concentration at one point divided by the time and spatially-averaged room concentration. Baughman et al. calculate the exposure index at each of the 41 points in the chamber for times extending from the moment the pollutant is first released. They show that for times less than or equal to τ_{mix}, the error in a single point measurement of exposure can be as much as 200−300% under quiescent conditions and as much as 100% when a heat source is present. However, exposure indices are only calculated for times lasting less than or equal to $\tau_{mix} = \tau_\alpha + \tau_\beta$ (31 min for quiescent conditions or 7 min when heat is added to the chamber). Consequently, it is difficult to know at exactly what time the time-averages at each point in the room approach the 41-point time-average. The time appears to be appreciably larger than τ_{mix}, but much less than τ_γ ($\sim$ 63−167 hrs).

Drescher et al. (9) conducted experiments similar to those by Baughman et al. (8), except under the more realistic conditions of forced mixing (using fans). They find mixing times of τ_{mix} = 2 to 15 min and, like Baughman et al., find that averaging times equal to τ_{mix} result in errors relative to the spatial room average of a factor of 2 or more. A detailed analysis showing what averaging times are sufficiently long to bring errors below 10% (or some other acceptable limit) is not provided.

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[1] In contrast, for the current paper we consider only τ_β as the "mixing time", since it gives an indication of how long an instantaneous release will take to become mixed under the given room conditions.

2.1 Discussion

The proportion of the time spent mixing relative to the source-off time as described by Mage and Ott – where τ_γ extends until the room concentration $z(t)$ reaches 1% of the peak concentration – gives a reasonable indicator of model accuracy. However, we require an estimate of the error associated with a given proportion. The two fundamental questions addressed in this paper are:

1. By how much can we truncate τ_γ for nearly-instantaneous sources, and still be confident that concentrations at a single point approximate the spatial room average over the entire study duration of $T = \tau_\alpha + \tau_\beta + \tau_\gamma$?
2. Given an acceptable τ_γ for nearly-instantaneous sources, how long must the study be to give acceptable results for continuous sources?

The studies considered so far only address the idealized case of nearly-instantaneous sources of ETS and do not determine errors in exposure for a range of time periods, i.e., times between the values of τ_{mix} and $\tau_{mix} + \tau_\gamma$. Real-life ETS exposures involve multiple ETS sources over the entire exposure duration. For example, how often do individuals remain in a room for a period of 4 hours during which only one cigarette has been smoked for 6 min (starting at the beginning of the time period)? In realistic situations, people will be present in bars, restaurants, cars, lounges, and offices in which, over the entire time period, either more than one person is smoking or one person is smoking multiple cigarettes. We should also consider realistic averaging times, which represent the average ETS exposure durations that people actually experience. For example, from the recent national human activity pattern study (1), we see that Americans are exposed to ETS on the order of $1-2$ hours in vehicles and bars or restaurants, and up to $5-8$ hours in residences and for persons working in offices and factories.

To evaluate the accuracy of the mass balance equation in estimating exposure of individuals to air pollutants, we must know the error associated with time-averaged concentrations predicted at a single point in a room over a wide range of averaging times. Specifically, we need to know the exposure duration at which the single-point, time-averaged exposure is within an acceptable error margin from the time and spatially-averaged room exposure. In the next section, I present data from several studies that provide an evaluation of error in exposure estimates under realistic conditions – including continuous ETS sources. Section 4 presents theoretical predictions of sufficient averaging times for continuous sources.

3 Error in Single-Point Exposure Estimates: Data from a Bedroom, a Tavern, and Smoking Lounges

How much error do we make when using a single-point concentration as a surrogate for the spatial room average? The error in a time-averaged exposure estimate or measurement in a room under conditions of non-ideal mixing (i.e., most real situations) is defined in this paper as the absolute difference between the time-averaged concentrations at a single point in the room and the time-averaged concentration of all points in the room. I will call this error the "exposure error" and consider it acceptable if it is less than 10% of the room average. The "relative error" is the exposure error divided by the time and spatially averaged room concentration. It is acceptable if it is under 0.10, i.e., if the exposure error is less than 10% of the room average. These errors depend on:

1. The amount of time required for mixing, here estimated for an instantaneous release as τ_β
2. The averaging time or exposure duration, T

The "mean exposure error" is the exposure error averaged over all monitored points. The "mean relative error" is the relative error averaged over all monitored points.

Different points in the room may have higher or lower exposure errors, but the mean exposure error should give a reasonable approximation of the error between a single point measurement and the exposure a person would receive at another point in the room or when moving about the room. The goal is to determine the required exposure duration T, which will result in mean relative errors lower than 10% for a room with a given post-source mixing time τ_β. To determine the study duration that gives mean relative errors under 10% across all points in the room, the running average pollutant concentration at different points must be calculated for extended time periods.

In several previous studies, three monitors were placed at widely separated points in a residential bedroom (10), a tavern (4), and smoking lounges (5, 11). These studies provide an opportunity to study real locations with progressively longer source-on times τ_α: a Marlboro cigarette was smoked for 6.5 min in the bedroom; four cigars were smoked for 11 min in the tavern; and smokers were constantly present in the lounge, providing a continuous ETS source.

Data from three monitors are not enough to fully analyze the distribution of exposures in the room at different times, but an estimate of the extreme exposures that could occur in each room can be provided by calculating the relative error for each point. These calculations have been performed and are reported here. By using running means over time, I determine the exposure duration that is required for the mean relative error to fall below 10% (see Table 1). This time ($\tau_{10\%}$) is the necessary exposure duration (starting just after the source becomes active) for measurements taken at one location to have an error margin of less than 10% relative to the spatial room mean. In other words, after $\tau_{10\%}$ minutes

Table 1: Summary of Results from Experiments Involving Three Widely-Spaced Monitors

Experiment Description	ϕ	τ	τ_α	τ_β	$^a\tau_{10\%}$
^{b}Residential Bedroom (CO; 1 Marlboro)	1.2/hr	50 min	6.5 min	$\sim$ 30 min	$\sim$ 15 min
^{c}Tavern (CO; 4 Cigars)	7.2/hr	8 min	11 min	$\sim$ 5 min	$\sim$ 12 min
^{d}Public Smoking Lounge (RSP; Multiple Smokers)	13/hr	4–5 min	continuous	–	$\sim$ 80 min
^{e}Company Smoking Lounge (Nicotine; Multiple Smokers)	13/hr	4–5 min	continuous	–	< 8 hr

$^a\tau_{10\%}$ is the time it takes after the tobacco source was ignited for the mean relative error of the three monitors to drop below approximately 10%. ^{b}Raw data are from Ott et al. (10). ^{c}Raw data are from Ott et al. (4). ^{d}Raw data are from Klepeis et al. (5). ^{e} Raw data are from Hammond (11). For sources of short duration, the time spent mixing (τ_β) was estimated to be the time that elapsed from the end of the source to when concentrations measured at the three monitors began to converge.

have passed, the exposure a person would experience at a given point in the room is, on average, only 10% different from the average room concentration.

3.1 Residential Bedroom

During a bedroom experiment (10), the 25.7 m^3 room had an air exchange rate of ϕ = 1.2/hr, which corresponds to a residence time of τ = 50 min. Carbon monoxide (CO) was measured at three points in the room: (1) a corner 5" from the floor; (2) the center of the room 36" from the floor; and (3) near the ceiling 95" from the floor. After being smoked from t = 0 min to t = 6.5 min, a Marlboro cigarette was extinguished and the CO levels decayed to background levels 7–8 hrs later. The time spent mixing (τ_β) was estimated at about 30 min. The mean relative error fell steadily for 5 min after the source started and dropped below 10% at approximately t = 15 min (see Figure 1).

3.2 Tavern

In a tavern experiment (4), the 521 m^3 room had an air exchange rate of about 7.2 ach with a residence time of about 8 min. As in the bedroom, CO was measured at three widely-spaced points in the room: (1) a central table, (2) a booth towards the southwest corner of the tavern, and (3) a booth in the northwest corner of the tavern. After four cigars were smoked two-at-a-time from t = 0 to t = 11 min, it took 40–45 min for the CO

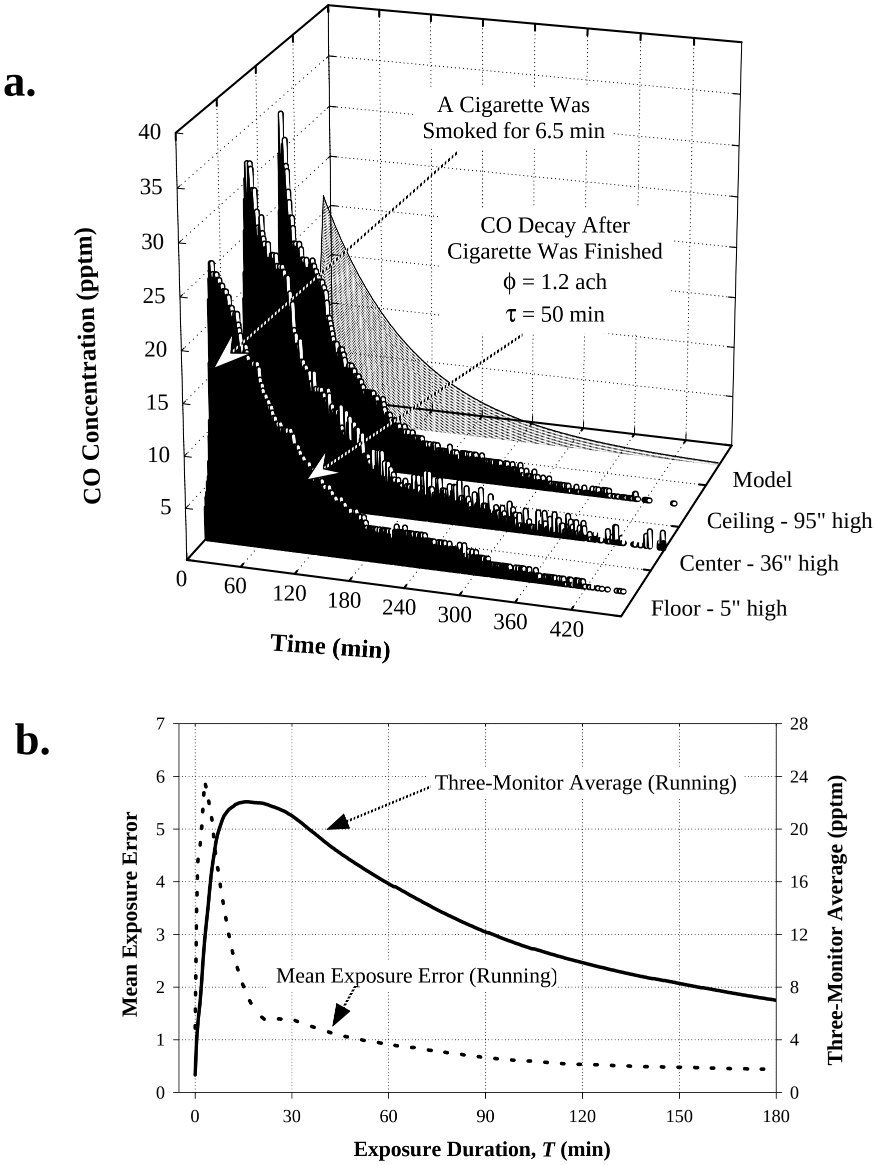

Figure 1: Panel (**a**) contains the carbon monoxide time series measured in three locations in a residential bedroom after a Marlboro cigarette was smoked from t = 0 to t = 6 min. Based on data from Ott et al. (10). Panel (**b**) contains the mean exposure error as a function of exposure duration, T, computed from the data in panel (**a**). Notice that after approximately t = 15 min, the mean relative error is less than 10%. Carbon monoxide concentrations are in units of parts per ten million (pptm).

levels to decay to their background level. The time segment τ_β was estimated at about 5 min. The mean relative error began a fairly steady decline starting at t = 12 min where it was 3%; it remained less than 5% thereafter (see Figure 2).

3.3 Smoking Lounges

Up to this point, I have considered single, relatively short point sources of air pollution: one or more cigarettes or cigars smoked over a time period on the order of 10 minutes at a single location in a room. A very different situation arises in smoking lounges, where cigarettes are continually smoked over extended time periods (a number of hours) and at multiple points throughout a room. Since it is difficult to obtain detailed information on the time and spatial coordinates of each cigarette that is smoked, investigators have treated multiple, overlapping cigarette sources as a single continuous source whose pollutant emission rate changes in time as the number of smokers present in the lounge changes (5). While it may be convenient at times (and even necessary) to equate a series of short sources with a single continuous source (see section 4, "Exposure to Continuous Sources" section), multiple point sources with an unknown spatial distribution can have an unpredictable affect on the mean relative error in exposure (see discussion below).

Hammond et al. (11) conducted a number of experiments in company smoking lounges in 1987 using three time-integrated nicotine samplers distributed throughout the lounge. The time period for three of the studies was 8 hours. The mean relative error for the three studies ranged from 0.5−2.5 %. The averaging time for which the mean relative error was 10% was most likely less than 8 hours, although we cannot pinpoint it exactly.

In one of 10 different experiments in airport smoking lounges (5), a 238 m^3 room had an air exchange rate of about 13 ach corresponding to a residence time of 4−5 min. Respirable suspended particles (RSP) were measured in a chair at the center of the room and in chairs at two opposite corners of the room starting at t = 0 min. There was an average of approximately 5 smokers present for the duration of the experiment and at least one smoker was present during each minute. The time spent mixing (τ_β) at this smoking lounge was probably similar to the mixing time in the tavern (about 5 min), since there were many people present who provided heat energy and a forced-air ventilation system was in operation. The mean relative error began to fall steadily at about t = 68 min and became 10% at about t = 80 min (see Figure 3).

Over all 10 smoking lounge experiments in which RSP was measured at three room locations, the mean relative error averaged out to 12% – ranging from 5% to 22%. The study time periods ranged from 60 min to 146 min. Regression results of model concentrations versus predicted concentrations were excellent for most of the study visits. The model prediction of the time-averaged room concentration matches the observed room average closely for all ten visits to smoking lounges.

As a caveat to the above calculations, note that mean relative error calculations that are based on only three monitoring positions are probably biased. In general, for both short

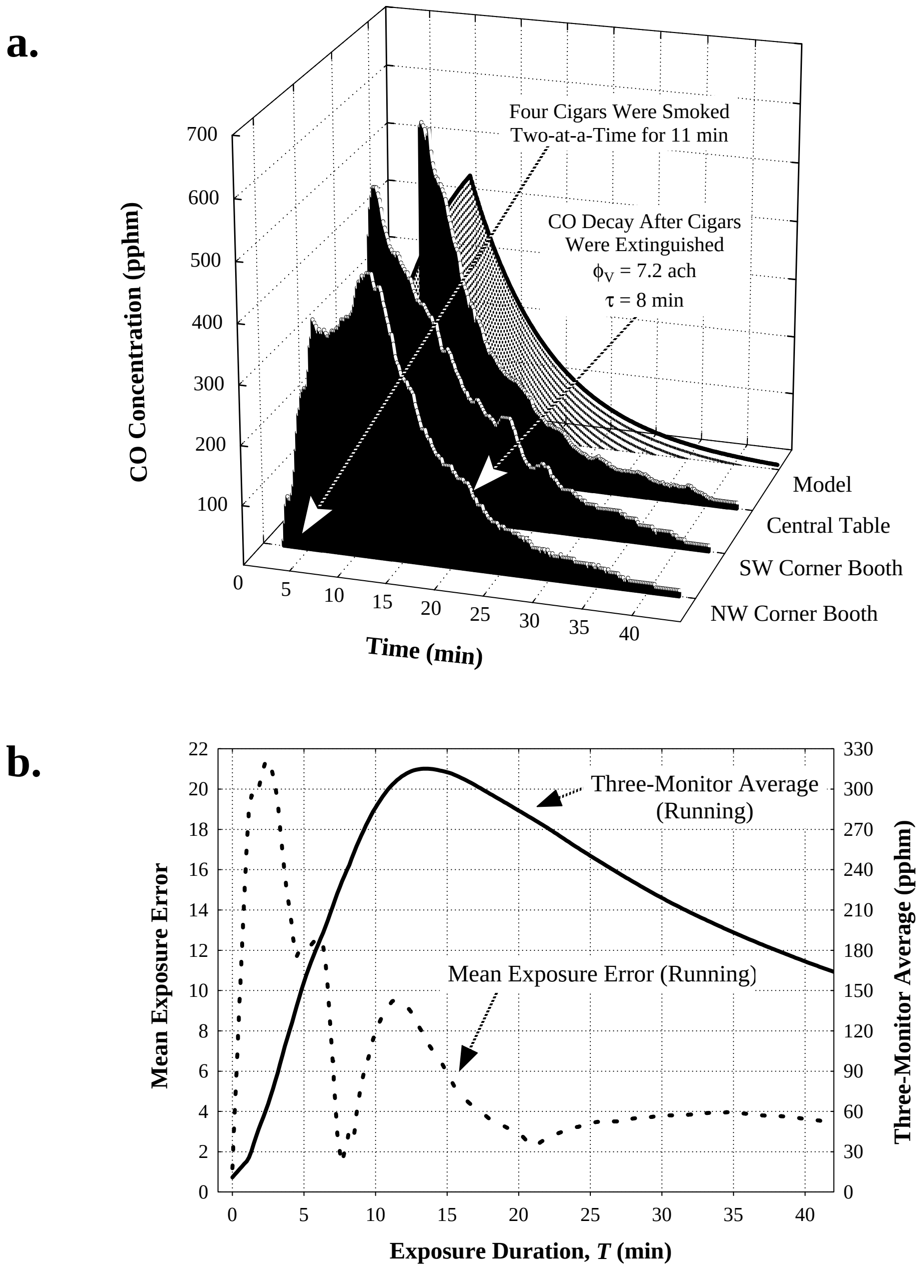

Figure 2: Panel (**a**) contains carbon monoxide time series measured in three locations in a 521 m^3 tavern after four cigars were smoked two-at-a-time from t = 0 to t = 11 min. Based on data from Ott et al. (4). Panel (**b**) contains the mean relative error as a function of exposure duration, T, computed from the data in panel (**a**). After t = 12 min, the mean relative error is consistently less than 10%, although it is also under 10% during parts of the smoking period. Carbon monoxide concentrations are in units of parts per hundred million (pphm).

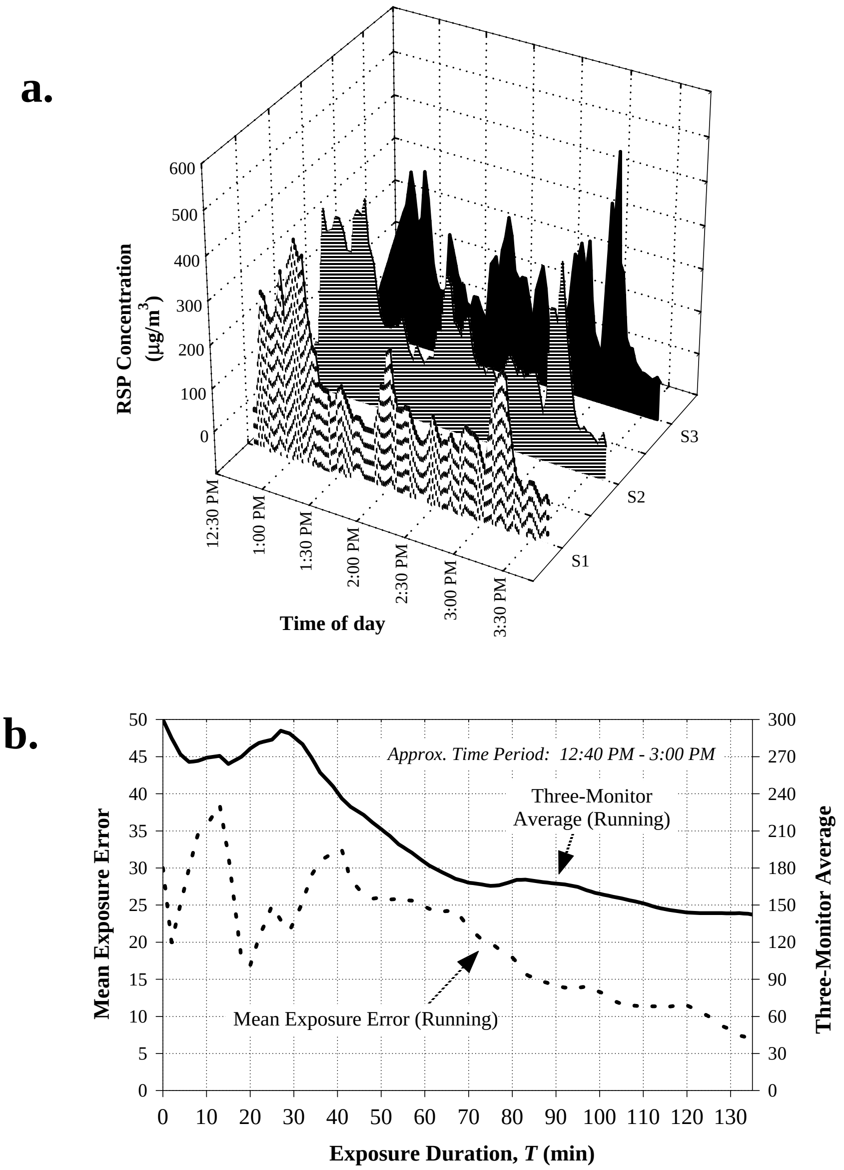

Figure 3: Panel (**a**) contains the respirable suspended particle (RSP) time series measured at three places in an enclosed airport smoking lounge where smokers were continuously present. Based on data from Klepeis et al. (5). Panel (**b**) contains the mean exposure error as a function of exposure duration, *T*, computed from the data in panel (**a**). The mean relative error decreases to 10% after an exposure duration of about 80 minutes. RSP concentrations are in units of micrograms per cubic meter.

and continuous sources, $\tau_{10\%}$, as determined from mean relative error calculations, can depend on the flow of air in the room, the direction of smoke emissions, the location of the smoker(s), and the emission rates of the different cigarettes. Without highly-resolved spatial monitoring, it is possible for error calculations to misrepresent the actual extent of mixing. For example, the range of mean relative error in smoking lounge studies may be a result of the location of smokers in each room. If smokers were fairly spread out in each lounge, or at least equal distances from each monitor, then the monitor concentrations could be fairly close to each other and to the room mean regardless of the rapidity of mixing.

3.4 Mobile Exposures

The fact that a person might be apt to move about in different locations of a room suggests that he/she could experience an exposure close to the average room concentration. Thus, it is possible that the true error in exposure for this person is smaller than that predicted for a single-point in the room by the mean relative error. For example, if we find that after a time T, the mean relative error in pollutant concentration across all points in the room is 9%, then the true error in relative exposure for a person moving about the room could be somewhat lower than 9%. It is impractical to consider occupant motion in exposure models, but we should keep in mind that the calculated mean exposure error could overestimate the true error under certain conditions. On the other hand, if a person spends too much time in close proximity to the source, the calculated mean exposure error could underestimate the true exposure. When dealing with the special case of continuous sources, Furtaw et al. (12) find that concentrations should be measured or estimated at distances of more than $0.4-0.8$ m from a source to minimize large positive errors.

4 Exposure to Continuous Sources

Mage and Ott (7), Baughman et al. (8), and Drescher et al. (9) consider sources of very short duration (τ_α = 20 s to 11 min) when compared to the entire exposure duration ($T = \tau_\alpha + \tau_\beta + \tau_\gamma > 1-2$ hr). For a single short source, after the source stops and the room has become well-mixed, the time series at the monitored locations have converged. This event occurs at the beginning of the τ_γ time period. The mean relative error falls within an acceptable error margin (arbitrarily chosen as 10%) beginning a time $\tau_{10\%}$ after the source has started.

As we have seen in the previous section, the time $t = \tau_{10\%}$ can occur either before or after the beginning of τ_γ. By completing measurements shortly after $t = \tau_{10\%}$ or at some later time, we are assured that the mean relative error will be less than 10%. However, many instances of human exposure to ETS can involve the presence of multiple smokers where smoke is constantly emitted into the room. In this case, multiple, overlapping smokers can be treated as a single continuous source. From results in smoking lounges (see previous section), it appears to take a much longer time for the mean concentrations at the three points to converge on the room mean (80 min in the smoking lounge vs. 5 min for fairly similar conditions in the tavern). Smoke emitted during any given short time interval may take only 5 min to become well-mixed, but an additional five minutes is needed for smoke emitted in each successive time interval to become well mixed. For continuous sources, the time series are constantly diverging, since smoke is continually emitted. How does the mean relative error ever reach an acceptable level?

Since our single-compartment mass balance equation is linear, it is possible to treat the time series arising from a single source as a superposition of a number of shorter sources (see Figure 4). I will call these shorter sources "sub-sources." For example, the pollutant time series of a 60-min source with a constant emission rate of 10 mg/min can be broken down into six identical sub-sources lasting 10 min each and each having a constant emission rate of 10 mg/min (see Figure 4a). The sub-sources can be considered as existing by themselves in separate rooms, with their starting points staggered by 10 min. The total time series is obtained by adding together all the time series in each of these rooms. For a source that has a varying emission rate or, equivalently, if there are different numbers of identical sub-sources present in each time interval (as in a smoking lounge where each cigarette is considered to have the same emission rate), the total pollutant time series can be treated as a number of staggered sub-sources with different emission rates. For example, the complicated pollutant time series of a 2-hr source can be broken down into twelve sub-sources lasting 10 min each with emission rates between 20 and 90 mg/min (see Figure 4b).

The sub-sources are assigned sub-durations (τ_{α}^{*}) that are very short when compared to the total exposure duration, T. A convenient value is δ, the time interval between measurements (e.g., $\delta = 1$ or $\delta = 10$ min for "real-time" monitoring). If each successive sub-source existed in the room by itself, it would have its own characteristic times τ_{β} and $\tau_{10\%}^{*}$. The duration of the sub-source time period is taken to be $T^{*} = \tau_{10\%}^{*}$, ending when the exposure error for the sub-source, taken by itself, is 10%.

Since the sub-sources are staggered in time, a number of the untruncated time periods, T^{*}, must be truncated so they will fit into the given total exposure duration T (see Figure 5). As T increases from zero, more and more of the untruncated sub-source time periods will fit into T, i.e., fewer and fewer need to be truncated in order to fit. When a large proportion of the complete time periods, T^{*}, fit (e.g., 90%), then we can be confident that the mean relative error of different monitors in the room will be close to 10%. For example, Figure 6 contains box plots for the distribution of sub-size time periods (either truncated or untruncated) over increasing values of the total study duration T. When the sub-source time period, T^{*}, is 10 min, it takes a total exposure duration of about $T = 100$ min before 90% of them fit without truncation (Figure 6a). This value of T is close to the value of 80 min that was reported for the smoking lounge experiment described above. When T^{*} is 30, about $T = 5$ hrs is required before 90% of the sub-sources fit without truncation (Figure 6b). This case corresponds to rooms with a very long τ_{β} value, such as under quiescent

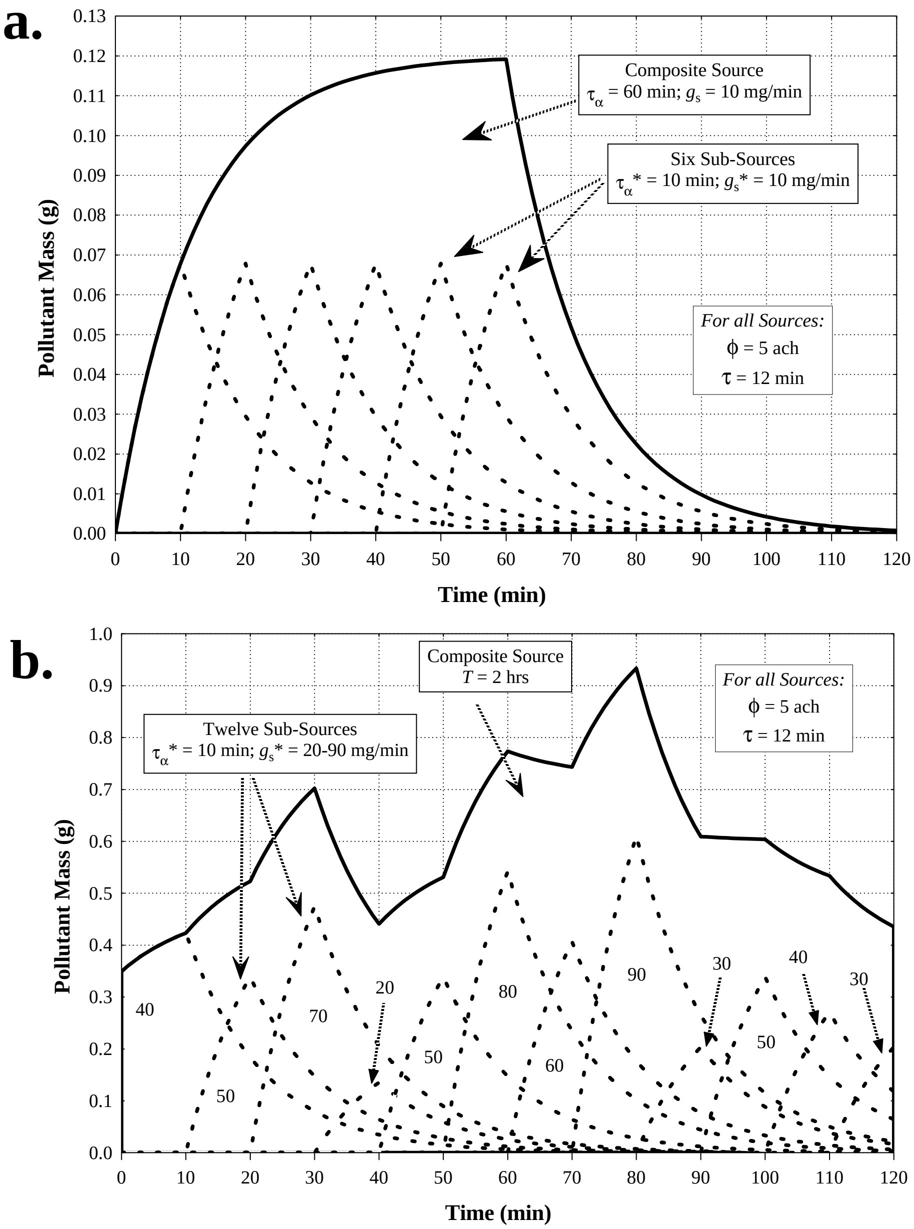

Figure 4: Pollutant mass time series illustrating the superposition of individual sub-sources into a single composite source. The air exchange rate was fixed at 5 ach. Dividing pollutant mass by the room volume gives the room concentration of the air pollutant. Panel (**a**) shows how a source with a constant emission rate ($g_s = 10$ mg/min) is broken down into six identical sub-sources lasting 10 min each and each having a $g_s^* = 10$ mg/min emission rate. Panel (**b**) shows how a source with a time-varying emission rate is broken down into twelve sub-sources lasting 10 min each and each having a constant source strength g_s^* from 20 to 90 mg/min.

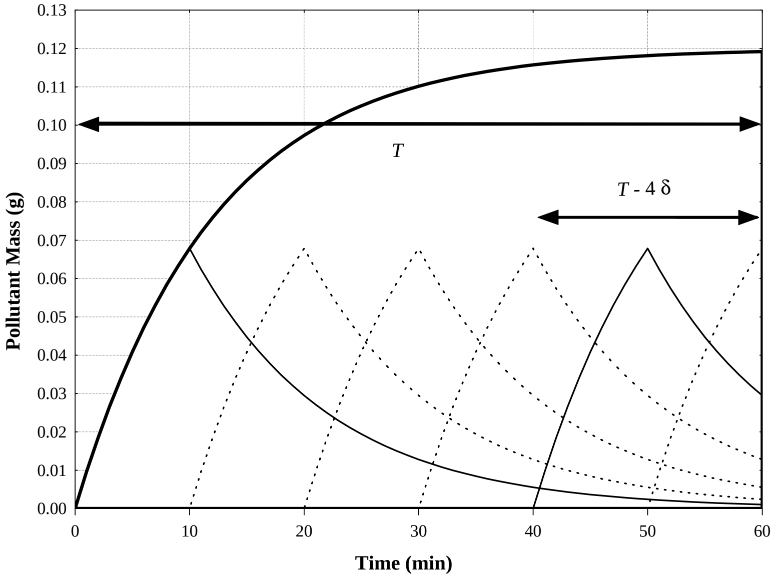

Figure 5: Plot showing how the sub-source time period T^* for early sub-sources can fit in the exposure time, T, whereas later sub-subsources are truncated. Notice that for T = 60 min, a sub-source duration, τ^*_α, of 10 min (same as δ, the study time resolution), and an untruncated sub-source time period, T^*, of, say, 30 min the first sub-source fits into T, but the fifth sub-source is truncated to T - 4δ.

conditions.

4.1 Summary

In the preceding section, I have provided a rough determination of the required averaging time for exposure studies involving continuous pollutant sources. A more thorough treatment might involve characterizing the movement of individual air parcels in a room, i.e., by using computational fluid dynamics (14). My method is based on the following relatively simple concept: If the mean relative exposure error associated with an arbitrarily short source is under 10% after some elapsed time, then the mean relative error for a series of these short sources is under 10% after a somewhat longer time. The short sources are staggered in time with one beginning immediately after another has ended so that their collective emissions are equivalent to the emissions from a single continuous source. We note that the mean relative error for continuous-source emissions only becomes acceptable after the mean relative errors for the bulk of the short sources are acceptable. This approach predicts that the time required for the mean relative error associated with the continuous source to fall below 10% is about 10 times the time that is required for a single short source.

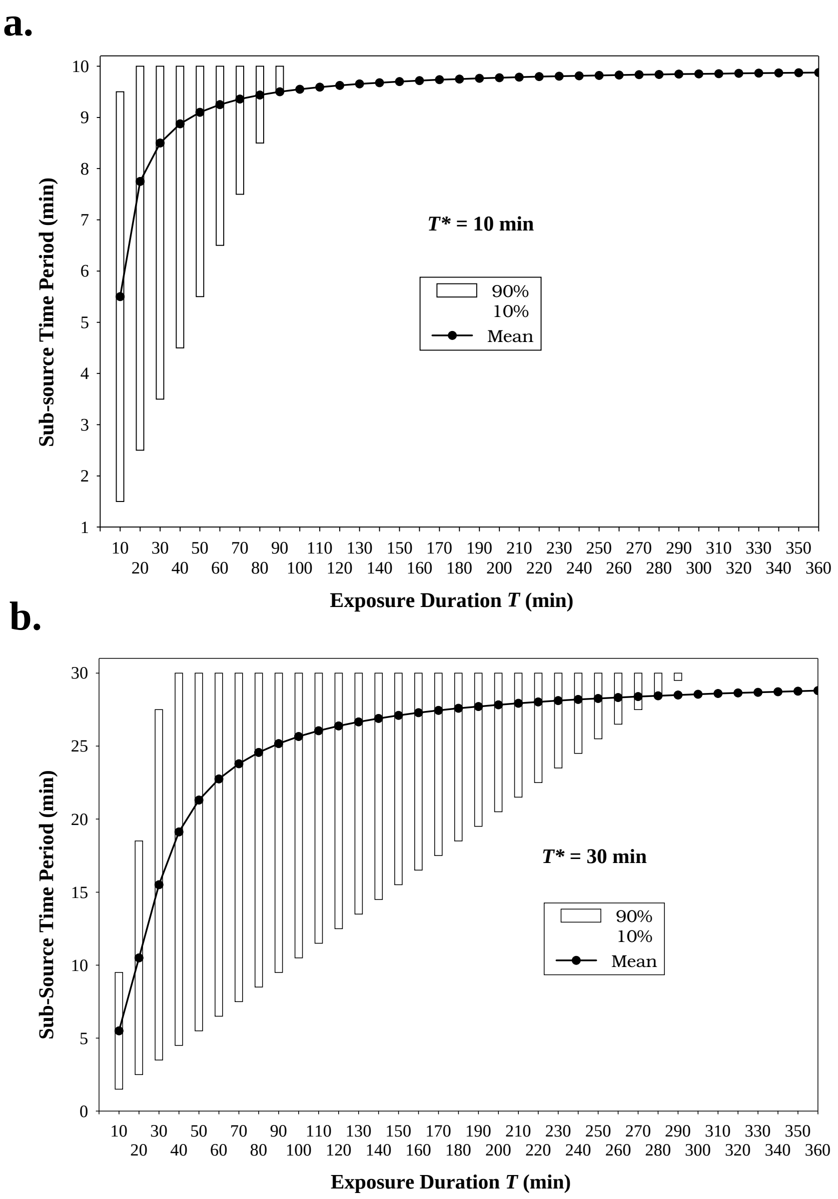

Figure 6: Boxplots showing the 90th and 10th percentiles of the distribution of the sub-source time period for a range of total study times and untruncated sub-source time periods, T^*. The untruncated sub-source time periods are T^* = 10 min for panel (**a**) and T^* = 30 min for panel (**b**). We consider the source to be continuous, and sub-sources nearly instantaneous (e.g., τ_α^* = 1 min) with respect to the total exposure duration. As stated in the text, the maximum, untruncated sub-source time period T^* is set to $\tau^*_{10\%}$, which is the time required so that the mean relative error is less than 10%. Notice that after a total exposure duration T of about 100 min, 90% of the complete, untruncated sub-source time periods T^* = 10 min fit into T, whereas for T^* = 30, it takes about 5 hrs.

5 Conclusions

How much error do we make when we use predictions or measurements of concentration at single points in a room as surrogates for an average room concentration? The exposure indices reported from two chamber studies indicate that times less than 7−31 min for natural convection and 2−15 min for forced convection are generally not long enough for the time-averaged room concentration at different points to approximate the time and spatially averaged room concentration. In contrast, results in a bedroom and a tavern suggest that under realistic conditions, averaging times on the order of 12−15 min may be long enough so that the exposure error is less than 10%. For a continuous source, the averaging time needs to be considerably longer. For conditions in an actual smoking lounge, an 80-min averaging time was required before the mean exposure error was less than 10%. From theoretical considerations, given adequate averaging times of 10 min and 30 min for a nearly-instantaneous source, adequate averaging times for a continuous source (under the same room conditions) are approximately ten times larger at 100 min and 5 hrs, respectively.

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